

On \mathbb{R}^3 , let ω_k denote a k -form. Then

$$T_1(d\omega_0) = \text{grad}(T_0(\omega_0)),$$

$$T_2(d\omega_1) = \text{curl}(T_1(\omega_1)),$$

$$T_3(d\omega_2) = \text{div}(T_2(\omega_2)).$$

$$T_0(\omega) = \omega$$

If $\omega = f_1(x, y, z)dx + f_2(x, y, z)dy + f_3(x, y, z)dz$, then $T_1(\omega) = (f_1, f_2, f_3)$

If $\omega = f_1(x, y, z)dx \wedge dy + f_2(x, y, z)dx \wedge dz + f_3(x, y, z)dy \wedge dz$, then $T_2(\omega) = (f_3, -f_2, f_1)$

If $\omega = f(x, y, z)dx \wedge dy \wedge dz$, then $T_3(\omega) = f(x, y, z)$

$$\omega_0 = f(x, y, z)$$

$$d\omega_0 = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial z} dz$$

$$T_1(d\omega_0) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)$$

$$T_0(\omega_0) = f(x, y, z)$$

$$\text{grad}(T_0(\omega_0)) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)$$

$$\omega_1 = f_1(x, y, z)dx + f_2(x, y, z)dy + f_3(x, y, z)dz$$

$$d\omega_1 = \left(\frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y} \right) dx \wedge dy + \left(\frac{\partial f_3}{\partial x} - \frac{\partial f_1}{\partial z} \right) dx \wedge dz + \left(\frac{\partial f_3}{\partial y} - \frac{\partial f_2}{\partial z} \right) dy \wedge dz$$

$$T_2(d\omega_1) = \left(\left(\frac{\partial f_3}{\partial y} - \frac{\partial f_2}{\partial z} \right), -\left(\frac{\partial f_3}{\partial x} - \frac{\partial f_1}{\partial z} \right), \left(\frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y} \right) \right)$$

$$T_1(\omega_1) = (f_1, f_2, f_3)$$

$$\text{curl}(T_1(\omega_1)) = \left(\left(\frac{\partial f_3}{\partial y} - \frac{\partial f_2}{\partial z} \right), -\left(\frac{\partial f_3}{\partial x} - \frac{\partial f_1}{\partial z} \right), \left(\frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y} \right) \right)$$

$$\omega_2 = f_1(x, y, z)dx \wedge dy + f_2(x, y, z)dx \wedge dz + f_3(x, y, z)dy \wedge dz$$

$$d\omega_2 = (f_1 - f_2 + f_3)dx \wedge dy \wedge dz$$

$$T_3(d\omega_2) = f_1 - f_2 + f_3$$

$$T_2(\omega_2) = (f_3, -f_2, f_1)$$

$$\text{div}(T_2(\omega_2)) = f_3 - f_2 + f_1$$