@Shi

1. 2 objects, elastic collision

find the final speed of the 2 objects

initial: $m_1$ with $v_{1i}$, $m_2$ with $v_{2i}$

final: $m_1$ with $v_{1f}$, $m_2$ with $v_{2f}$

known: $m_1$, $m_2$, $v_{1i}$, $v_{2i}$

to find: $v_{1f}$, $v_{2f}$

two equations to solve two unknowns

conservation of momentum

$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$ ①

conservation of (kinetic) energy

$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$ ②

there are quadratic terms that you may find it hard to deal with, if you start to plug in the actual values of ($m_1$, $m_2$, $v_{1i}$, $v_{2i}$) to get ($v_{1f}$, $v_{2f}$)

Note that ① & ② are two pieces of info you can re-arrange ① & ② to get ③ then use ① & ③ or ② & ③, instead of ① & ②

To simplify, cz ② is quadratic

Can we get rid of ② and use a simple ③ & ①

to solve the question?

from ① $\Rightarrow \underbrace{m_1 (v_{1i} - v_{1f})}_{a} = \underbrace{m_2 (v_{2f} - v_{2i})}_{b}$

from ② $\Rightarrow \underbrace{\frac{1}{2} m_1 (v_{1i} - v_{1f})(v_{1i} + v_{1f})}_{c} = \underbrace{\frac{1}{2} m_2 (v_{2f} - v_{2i})(v_{2f} + v_{2i})}_{d}$

use $\frac{c}{a} = \frac{d}{b} \Rightarrow v_{1i} + v_{1f} = v_{2f} + v_{2i}$ ③ yes!

To find $v_{1f}$

from ③, $v_{2f} = v_{1i} + v_{1f} - v_{2i}$

into ①, $m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 (v_{1i} + v_{1f} - v_{2i})$

$\Rightarrow v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}$

To find $v_{2f}$

from ③, $v_{1f} = v_{2i} + v_{2f} - v_{1i}$

into ①, $m_1 v_{1i} + m_2 v_{2i} = m_1 (v_{2i} + v_{2f} - v_{1i}) + m_2 v_{2f}$

$\Rightarrow v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}$

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2. Elastic collision with a wall

before: $m$ with $v$

after: $m$ with $-v$

velocity of wall

$v_{wi} = 0$

$v_{wf} \simeq 0$

"w" is for "wall"!

mass of wall: $M \to \infty$

Question: elastic collision means conservation of both kinetic energy and momentum.

the momentum of the object is $mv$ and $(-mv)$ before and after, with the wall not moving (momentum = 0)

It seems violation of conservation of momentum! Well →

Conservation of momentum

$\underbrace{mv_i}_{\text{which we think is } v} + 0 = \underbrace{mv_f}_{\text{which we think is } -v} + Mv_{wf}$ ①

conservation of kinetic energy

$\frac{1}{2} mv_i^2 + 0 = \frac{1}{2} mv_f^2 + \frac{1}{2} Mv_{wf}^2$ ②

just assume that $Mv_{wf}$ can compare with $mv_i$, but with $M \gg m$, $v_{wf} \ll v_i$

So it is safe to take $\frac{1}{2} Mv_{wf}^2 \ll \frac{1}{2} mv_i^2$

thus you can write ② as $\frac{1}{2} mv_i^2 + 0 = \frac{1}{2} mv_f^2 + 0^+$

then $v_i^2 = v_f^2$, before & after collision

you take $v_f = -v_i$

⇒ that's why the amplitude doesn't change with direction opposite

take $v_f = -v_i$ into ①

$mv_i + 0 = m(-v_i) + Mv_{wf}$

Although $v_{wf} \ll v_i$, you can assume that "the wall doesn't move"

but the momentum it gains ($Mv_{wf}$) is not neglectable!

Actually $Mv_{wf} \simeq 2mv_i$

Obviously, it is not safe to take $mv_i + 0 = mv_f + \underbrace{0^+}_{\text{not small}}$