How a non-equilibrium system evolve over time

Theory of diffusion

2 assumptions

1. A substance will move down to its concentration gradient (Fick's first law)

$\underset{\text{flux}}{\vec{J}} = -\underset{\substack{\text{diffusion}\\ \text{constant}}}{D}\nabla C \rightarrow$ concentration 1D: $J = -D\frac{\partial C}{\partial x}$

2. conservation of matter

$\frac{\partial C}{\partial t} = -\underset{\text{divergence}}{\vec{\nabla}\cdot}\vec{J}$ 1D: $\frac{\partial C}{\partial t} = -\frac{\partial J}{\partial x}$

All togather, we get Diffusion equation

$\frac{\partial C}{\partial t} = D\underset{\substack{\text{Laplacian}\\ \text{operator}}}{\nabla^2} C$ 1D: $\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2}$

Diffusion equation: a partial differential equation

how a spatial distribution of material changes over time

Define solution $\begin{cases}\text{initial conditions}\\ \text{boundary conditions}\end{cases}$

One-dimensional diffusion from a point

Initial conditions: $C = M\underset{[L]^{-1}}{\underbrace{\delta(x)}}$ @ $t=0$ which means $\int_{-\infty}^{\infty} C(x)dx = M$

boundary conditions: Length $= \infty$

Brown part: If not familiar with complex form of Fourier transformation

Periodic functions: expand to Fourier series

Non-periodic functions: functions of $\underline{\infty \text{ period}}$ $\rightarrow$ Fourier integrals

frequency: $0 \rightarrow \infty$

$F(x) = \int_0^{\infty} A(s)\sin(sx)ds + \int_0^{\infty} B(s)\cos(sx)ds$ where $\begin{cases} A(s) = \frac{1}{\pi}\int_{-\infty}^{\infty} F(x)\sin(sx)dx \\ B(s) = \frac{1}{\pi}\int_{-\infty}^{\infty} F(x)\cos(sx)dx \end{cases}$

$\begin{cases} \cos(sx) = \frac{1}{2}(e^{ixs} + e^{-ixs}) \\ \sin(sx) = \frac{1}{2i}(e^{ixs} - e^{-ixs}) \end{cases}$

$F(x) = \int_0^{\infty} \frac{1}{2}[B(s) - iA(s)]e^{ixs}ds + \underline{\int_0^{\infty} \frac{1}{2}[B(s) + iA(s)]e^{-ixs}ds}$

$\downarrow$

$\int_{-\infty}^{0} \frac{1}{2}[A(|w|) + iB(|w|)]e^{isx}ds$

set $G(s) = \begin{cases} \frac{1}{2}[B(w) - iA(w)] & (s \geq 0) \\ \frac{1}{2}[B(|w|) + iA(|w|)] & (s < 0) \end{cases}$

$F(x) = \int_{-\infty}^{\infty} G(s)e^{ixs}ds$ transformation between two domains: (x, s)

typo: P479

(A3.10) should be

$\int_{-\infty}^{\infty} F(x)e^{-ixs'}dx = 2\pi\int_{-\infty}^{\infty} G(s)\delta(s-s')ds$

Fourier integrals in complex form

$F(x) = \int_{-\infty}^{\infty} G(s)e^{ixs}ds$

For $s \geq 0$, $G(s) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(x)[\cos(sx) - i\sin(sx)]dx$

$= \frac{1}{2\pi}\int_{-\infty}^{\infty} F(x)e^{-ixs}dx$

For $s < 0$, $G(s) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(x)[\cos|s|x + i\sin(|s|w)]dx$

$= \frac{1}{2\pi}\int_{-\infty}^{\infty} F(x)e^{ix|s|}dx$

$= \frac{1}{2\pi}\int_{-\infty}^{\infty} F(x)e^{-ixs}dx$

$G(s) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(x)e^{-ixs}dx$