🥺

$$P(x=m;t_c=0)=\sum_{n=m}^{\infty}\binom{n}{m}\left(\frac{1}{2}\right)^n P(x=n;t_c=T)$$

$$G(z;t_c)=\sum_{m=0}^{\infty} z^m P(x=m;t_c)$$ 唯一对称

$$G(z;t_c=0)=\sum_{m=0}^{\infty} z^m \sum_{n=m}^{\infty}\binom{n}{m}\left(\frac{1}{2}\right)^n P(x=n;t_c=T)$$

$$=\sum_{n=0}^{\infty}\sum_{m=0}^{n}\binom{n}{m}\left(\frac{1}{2}\right)^{n-m}\left(\frac{z}{2}\right)^m P(x=n;T)$$

$$=\sum_{n=0}^{\infty}\left(\frac{1+z}{2}\right)^n P(x=n;T)$$

$$=G\left(\frac{1+z}{2};T\right)$$

First moment: Mean

$$\left.\frac{\partial G(z,t_c)}{\partial z}\right|_{z=1}=\langle x(t_c)\rangle$$

$$\langle x(0)\rangle=\left.\left(\frac{\partial G(z,t_c)}{\partial z}\right)\right|_{z=1}$$

$$=\left.\left(\frac{\partial G\left(\frac{1+z}{2};T\right)}{\partial z}\right)\right|_{z=1}$$

$$=\frac{1}{2}\left.\left(\frac{\partial G(z;T)}{\partial z}\right)\right|_{z=1}$$

$$=\frac{1}{2}\langle x(T)\rangle$$

$$\langle x(T)\rangle=2\langle x(0)\rangle$$

Second moment:

$$\left.\frac{\partial^2 G(z,t_c)}{\partial z^2}\right|_{z=1}=\langle x^2(t_c)\rangle-\langle x(t_c)\rangle$$

Raw moment:

$$\langle x^2(t_c)\rangle=\left.\left(\frac{\partial^2 G(z,t_c)}{\partial z^2}+\frac{\partial G(z,t_c)}{\partial z}\right)\right|_{z=1}$$

$$\langle x^2(0)\rangle=\left.\left(\frac{\partial^2 G(z,0)}{\partial z^2}+\frac{\partial G(z,0)}{\partial z}\right)\right|_{z=1}$$

$$=\left.\left(\frac{\partial^2 G\left(\frac{1+z}{2};T\right)}{\partial z^2}+\frac{\partial G\left(\frac{1+z}{2};T\right)}{\partial z}\right)\right|_{z=1}$$

$$=\left.\left(\frac{1}{4}\frac{\partial^2 G(z;T)}{\partial z^2}+\frac{1}{2}\frac{\partial G(z;T)}{\partial z}\right)\right|_{z=1}$$

$$=\frac{1}{4}\langle x^2(T)\rangle-\frac{1}{4}\langle x(T)\rangle+\frac{1}{2}\langle x(T)\rangle$$

$$=\frac{1}{4}\langle x^2(T)\rangle+\frac{1}{4}\langle x(T)\rangle$$

$$\langle x^2(T)\rangle=4\langle x^2(0)\rangle-2\langle x(0)\rangle$$

Central moment: Variance:

$$\sigma_x^2(t_c)=\left.\left(\frac{\partial^2 G(z,t_c)}{\partial z^2}+\frac{\partial G(z,t_c)}{\partial z}+\left(\frac{\partial G(z,t_c)}{\partial z}\right)^2\right)\right|_{z=1}$$

$$\sigma_x^2(0)=\left.\left(\frac{\partial^2 G(z,0)}{\partial z^2}+\frac{\partial G(z,0)}{\partial z}+\left(\frac{\partial G(z,0)}{\partial z}\right)^2\right)\right|_{z=1}$$

$$=\left.\left(\frac{1}{4}\frac{\partial^2 G(z,T)}{\partial z^2}+\frac{1}{2}\frac{\partial G(z;T)}{\partial z}+\frac{1}{4}\left(\frac{\partial G(z,T)}{\partial z}\right)^2\right)\right|_{z=1}$$

$$=\left.\left(\frac{1}{4}\frac{\partial^2 G(z,T)}{\partial z^2}+\frac{1}{4}\frac{\partial G(z;T)}{\partial z}+\frac{1}{4}\left(\frac{\partial G(z,T)}{\partial z}\right)^2+\frac{1}{4}\frac{\partial G(z;T)}{\partial z}\right)\right|_{z=1}$$

$$=\frac{1}{4}\sigma_x^2(T)+\frac{1}{4}\langle x(T)\rangle$$

$$\sigma_x^2(T)=4\sigma_x^2(0)-2\langle x(0)\rangle$$