

Eq. (1)

$$\frac{\partial p(x)}{\partial t} = \frac{\partial}{\partial x} [\gamma_2 x p(x)] + k_1 \int_0^x dx' w(x-x') p(x')$$

where  $w(x, x') = w(x-x') = \nu(x-x') - \delta(x-x')$

$$\frac{\partial p(x)}{\partial t} = 0 \Rightarrow -\frac{\partial}{\partial x} [x p(x)] = \underbrace{a}_{a = \frac{k_1}{\gamma_2}} w * p(x)$$

$$\xrightarrow{\text{Laplace}} s \frac{\partial \hat{p}(s)}{\partial s} = a \hat{w}(s) \hat{p}(s)$$

$\nu(x) = \frac{1}{b} \exp(-\frac{x}{b})$  — exponential distribution

$$\hat{w}(s) = \int_0^{\infty} (\nu(x) - \delta(x)) e^{-sx} dx$$

$$= \int_0^{\infty} (\frac{1}{b} \exp(-\frac{x}{b}) e^{-sx} - \delta(x) e^{-sx}) dx$$

$$= \frac{1}{b} \frac{1}{s + \frac{1}{b}} (0-1) - 1$$

$$= \frac{1}{b} \frac{1}{s + \frac{1}{b}} - \frac{sb+1}{b(s+\frac{1}{b})}$$

$$= -\frac{s}{s + \frac{1}{b}}$$

$$s \frac{\partial \hat{p}}{\partial s} = a \hat{w} \hat{p}$$

$$\frac{\partial \hat{p}}{\hat{p}} = a \hat{w} \frac{\partial s}{s}$$

$$\ln \hat{p} = a \frac{-s}{s + \frac{1}{b}}$$

$$\hat{p}(0) = 1 \Rightarrow \hat{p}(s) = (s + \frac{1}{b})^{-a}$$

Eq. (b)

$$-\frac{\partial}{\partial x} [x p(x)] = a \int_0^x dx' w(x-x') c(x') p(x')$$

$$\xrightarrow{\text{Laplace}} s \frac{\partial \hat{p}}{\partial s} = a \left( \frac{-s}{s + \frac{1}{b}} \right) \hat{c} * \hat{p}$$

$$= a \left( \frac{-s}{s + \frac{1}{b}} \right) (\hat{c} * \hat{p})$$

$$\frac{\partial \hat{p}}{\partial s} = a \left( \frac{-1}{s + \frac{1}{b}} \right) (\hat{c} * \hat{p})$$

$$\frac{x(s + \frac{1}{b})}{s} \frac{\partial \hat{p}}{\partial s} + \frac{1}{b} \frac{\partial \hat{p}}{\partial s} = -a \hat{c} * \hat{p}$$

$$\xrightarrow{\text{Laplace}} -\frac{\partial}{\partial x} [x p(x)] - \frac{1}{b} x p = -a c p$$

$$\frac{\partial}{\partial x} (x p) + \frac{x p}{b} = a c p$$